AIAA JOURNAL
Vol. 41, No. 9, September 2003

Functionally Graded Cylindrical Shell Thermal Instability Based on Improved Donnell Equations

R. Shahsiah*
Azad University, 14515-775 Tehran, Iran
and
M. R. Eslami†
Amirkabir University of Technology, 15914 Tehran, Iran

The thermal instability of cylindrical shells of functionally graded material is considered. The derivation of equations is based on first-order shell theory and the complete Sanders kinematic equations. The resulting equilibrium and the stability equations contain the rotations in the x and θ directions and the transverse shear force in the θ direction, in addition to the conventional Donnell equations. When it is assumed that the material properties vary linearly through the thickness direction, the system of fundamental partial differential equations in terms of the displacement components is established. Instability analysis of functionally graded cylindrical shells under two types of thermal loads with simply supported boundary conditions is carried out. Results are obtained in analytical form. The results are validated with the known data in the literature.

Nomenclature

E_m, E_c	=	modulus of elasticity of metal and ceramic
f_c	=	volume fraction of ceramic
f_m	=	volume fraction of metal
h	=	thickness of cylindrical shell
k_{ij}	=	curvature of middle surface
M_{ij}	=	moment resultant
$(M_{ij})_1$	=	moment resultant of stable state
N_{ij}	=	force resultant
$(N_{ij})_1$	=	force resultant of stable state
R	=	mean radius of cylindrical shell
u	=	axial displacement
u_1, v_1, w_1	=	displacement components of stable state
v	=	circumferential displacement
w	=	lateral displacement
α_m, α_c	=	thermal expansion coefficient of metal and ceramic
β_x	=	rotation of middle surface about the x direction
β_{x_1}	=	rotation of stable state about the x direction
β_θ	=	rotation of middle surface about the θ direction
β_{θ_1}	=	rotation of stable state about the θ direction
$(\gamma_{ij})_m$	=	shear strain of middle surface
$(\gamma_{ij})_{m1}$	=	shear strain of middle surface of stable state
ϵ_{ii}	=	normal strain
$(\epsilon_{ii})_m$	=	normal strain of middle surface
$(\epsilon_{ii})_{m1}$	=	normal strain of middle surface of stable state
ν	=	Poisson's ratio
σ_{ii}	=	normal stress
τ_{ij}	=	shear stress

I. Introduction

RECENT studies on new performance materials have been extended to a new material known as functionally graded materials (FGMs). These are high-performance heat resistant materials able to withstand ultrahigh temperatures and extremely large thermal gradients used in the aerospace industries. FGMs are microscopically inhomogeneous with mechanical properties that vary smoothly and continuously from one surface to the other.[1] Typically, these materials are made from a mixture of ceramic and metal.

Generally, there are two ways to model the material property gradation in solids: 1) Assume a profile for volume fraction and 2) use a micromechanics approach to study the nonhomogeneous media. For composition profile modeling, polynomial representations including quadratic[2] and cubic[3,4] variations are used. Other representations, such as exponential functions[5,6] and piecewise homogeneous layer representations[7,8] have also been used. At the microstructural level, a FGM is characterized by transition from a dispersive phase to an alternative structure with a networking structure in between. Zhai et al.[9] and Nan et al.[10] directly address the constitutive relations of FGMs. Specifically, Nan et al.[10] used an analytical approach to describe the uncoupled thermomechanical properties of metal/ceramic FGMs. Pindera and Freed,[11] Pindera et al.,[12] and Aboudi et al.[13] used the unit-cell approach to analyze FGMs. A recent special issue on FGMs[14] focuses primarily on the micromechanics-based studies.

Thermal buckling analysis of perfect cylindrical shells of isotropic and homogeneous materials and cylindrical shells of composite materials based on the Donnell and improved Donnell stability equations are studied by Eslami et al.[15] and Eslami and Javaheri.[16] Eslami and Shariyat[17] considered the flexural theory and, with the full Green nonlinear strain–displacement relations instead of the simplified Sanders assumption, formulated the dynamic mechanical and thermal buckling of imperfect cylindrical shells. The higher-order shear deformation theory, including the normal stress, was used, and the mixed formulation was established to simplify the approach of both kinematic and forced boundary conditions. The technique was then improved by the same authors to an exact three-dimensional analysis of circular cylindrical shells based on the equilibrium equation and the full nonlinear Green strain-displacement relations.[18] The Donnell and improved Donnell stability equations are employed to present a closed-form solution for the elastoplastic and creep buckling of cylindrical shells under mechanical loads at an elevated temperature.[19] Eslami and Shahsiah determined the critical thermal buckling loads for imperfect cylindrical shells.[20] They used the Donnell and the improved Donnell stability equations and two models for imperfection, namely, the Wan–Donnell and Koiter models. Many postbuckling studies based on the classical shell theory of composite laminated thin cylindrical shells subjected to mechanical or thermal loading or their combinations are available in the literature, such as those of Birman and Bert[21] and Shen.[22–24] Relatively few studies involve the application

Received 30 May 2002; revision received 14 April 2003; accepted for publication 15 April 2003.

*Ph.D. Student, Mechanical Engineering Department, Science and Research Campus.

†Professor, Mechanical Engineering Department, Distingiushed Solid Mechanics Center. Associate Fellow AIAA.

of shear deformation shell theory to the postbuckling analysis in the literature, such as those given by Iu and Chia[25] Reddy and Savoia.[26] In these studies, the material properties are considered to be independent of temperature. However, studies of temperature and moisture effects on the buckling loads of laminated flat and cylindrical panels are limited in number,[27–31] and all of these studies assumed a perfect initial configuration. Palazotto and Tisler,[32] Palazotto,[33] Horban and Palazotto,[34] Siefert and Palazotto,[35] Dennis and Palazotto,[36,37] Isai et al.,[38] Dennis and Palazotto,[39] Schimmels and Palazotto,[40] Palazotto et al.,[41] Chien and Palazotto,[42] and Schimmels and Palazotto[43] have done extensive theoretical and experimental work on the stability of composite panels. Their work substantially reduced the gap between the theoretical and experimental works. Shen[44] gave a full nonlinear postbuckling analysis of composite laminated cylindrical shells subjected to combined loading of axial compression and external pressure under hygrothermal conditions.

Buckling analyses of FGM structures are rare in the literature. Birman[45] studied the buckling problem of functionally graded composite rectangular plate subjected to the uniaxial compression. The stabilization of a functionally graded cylindrical shell under axial harmonic loading was investigated by Ng et al.[46] Javaheri and Eslami presented the thermal and mechanical buckling of rectangular FGM plates based on the first- and higher-order plate theories.[47–50] The buckling analysis of circular FGM plates is given by Najafizadeh and Eslami.[51]

In this paper, the thermal instability of functionally graded cylindrical shell is considered. The complete Sanders nonlinear strain–displacement relations are used. The resulting equilibrium and stability equations contain the x and θ components of the rotation and the transverse shear force in the θ direction, in addition to the conventional Donnell equations. This approximation provides more reliable results for the cylindrical shells of relatively longer lengths. It is assumed that the cylindrical shell is under uniform temperature rise and radial temperature difference for thermal loading. Simply supported boundary conditions are assumed.

II. Derivations

Consider a cylindrical shell made of FGM. The shell is assumed to be graded through the thickness direction. The constituent materials are assumed to be ceramic and metal. The volume fractions of ceramic f_c and metal f_m corresponding to the power law are expressed as[52]

$$f_c = [(2z + h)/2h]^k, \qquad f_m = 1 - f_c \tag{1}$$

where z is the thickness coordinate, $-h/2 \le z \le h/2$, h is the thickness of the shell, and k is the power law index, which takes values greater than or equal to zero.[52] In this paper, it is assumed that the variation of the composition of ceramic and metal is linear, $k = 1$. The justification for this assumption is given in Appendices A and B. The value of k equal to zero represents a fully ceramic shell. The mechanical and thermal properties of FGMs are determined from the volume fraction of the material constituents. We assume that the nonhomogeneous material properties such as the modulus of elasticity E and the coefficient of thermal expansion α change in the thickness direction z based on the Voigt's rule over the whole range of volume fraction (see Ref. 53) as

$$E(z) = E_c f_c + E_m(1 - f_c)$$

$$\alpha(z) = \alpha_c f_c + \alpha_m(1 - f_c), \qquad \nu(z) = \nu \tag{2}$$

where subscripts m and c refer to the metal and ceramic constituents, respectively. When volume fractions are substituted from Eqs. (1) in Eqs. (2), material properties of the FGM shell are determined, which are the same as the equations proposed by Praveen and Reddy[52] as

$$E(z) = E_m + E_{cm}[(2z + h)/2h]$$

$$\alpha(z) = \alpha_m + \alpha_{cm}[(2z + h)/2h], \qquad \nu(z) = \nu \tag{3}$$

where

$$E_{cm} = E_c - E_m, \qquad \alpha_{cm} = \alpha_c - \alpha_m \tag{4}$$

Consider a thin cylindrical shell of mean radius R and thickness h with length L. The normal and shear strains at distance z from the shell middle surface are[54]

$$\epsilon_x = \epsilon_{xm} + zk_x, \qquad \epsilon_\theta = \epsilon_{\theta m} + zk_\theta, \qquad \gamma_{x\theta} = \gamma_{x\theta m} + zk_{x\theta} \tag{5}$$

where ϵ_{ij} is the normal strain, γ_{ij} is the shear strain, and k_{ij} is the curvature. The subscript m refers to the strain at the middle surface of the shell. The indices x and θ refer to the axial and circumferential directions, respectively. The nonlinear strain–displacement relations according to the complete Sanders assumption (see Ref. 54) are

$$\epsilon_{xm} = u_{,x} + \tfrac{1}{2}w_{,x}^2, \qquad \epsilon_{\theta m} = (v_{,\theta} + w)/R + (v - w_{,\theta})^2\big/2R^2$$

$$\gamma_{x\theta m} = u_{,\theta}/R + v_{,x} + w_{,x}(w_{,\theta} - v)/R, \qquad k_x = -w_{,xx}$$

$$k_\theta = (v_{,\theta} - w_{,\theta\theta})\big/R^2, \qquad k_{x\theta} = (v_{,x} - 2w_{,x\theta})/2R \tag{6}$$

where u, v, and w are the axial and circumferential displacements and the lateral deflections of the shell, respectively, and the subscript comma indicates a partial derivative. Terms $v_{,\theta}$ and $v_{,x}$ in the fifth and sixth of Eqs. (6) are eliminated in the classical derivations of the Donnell equilibrium and the stability equations. Hooke's law for a functionally graded cylindrical shell is defined as[1]

$$\sigma_x = \frac{E(z)}{1-\nu^2}(\epsilon_{xm} + zk_x + \nu\epsilon_{\theta m} + \nu z k_\theta) - \frac{E(z)\alpha(z)T}{1-\nu}$$

$$\sigma_\theta = \frac{E(z)}{1-\nu^2}(\epsilon_{\theta m} + zk_\theta + \nu\epsilon_{xm} + \nu z k_x) - \frac{E(z)\alpha(z)T}{1-\nu}$$

$$\tau_{x\theta} = G(z)(\gamma_{x\theta m} + zk_{x\theta}) \tag{7}$$

where $E(z)$ and $\alpha(z)$, are the elastic modulus and thermal expansion coefficient.

The force and moment resultants expressed in terms of the stress components through the thickness, according to first-order shell theory, are

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\,\mathrm{d}z, \qquad M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z\,\mathrm{d}z \tag{8}$$

Substituting Eqs. (3) and (7) in Eqs. (8) results in the constitutive law in terms of the displacement components as

$$N_x = \beta_1\epsilon_{xm} + \beta_1\nu\epsilon_{\theta m} + \beta_2 k_x + \beta_2\nu k_\theta - \beta_3/(1-\nu)$$

$$N_\theta = \beta_1\epsilon_{\theta m} + \beta_1\nu\epsilon_{xm} + \beta_2 k_\theta + \beta_2\nu k_x - \beta_3/(1-\nu)$$

$$N_{x\theta} = \beta_4\gamma_{x\theta m} + \beta_5 k_{x\theta}$$

$$M_x = \beta_2\epsilon_{xm} + \beta_2\nu\epsilon_{\theta m} + \beta_6 k_x + \beta_6\nu k_\theta - \beta_7/(1-\nu)$$

$$M_\theta = \beta_2\epsilon_{\theta m} + \beta_2\nu\epsilon_{xm} + \beta_6 k_\theta + \beta_6\nu k_x - \beta_7/(1-\nu)$$

$$M_{x\theta} = \beta_5\gamma_{x\theta m} + \beta_8 k_{x\theta} \tag{9}$$

where

$$\beta_1 = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2}\,\mathrm{d}z, \qquad \beta_2 = \int_{-h/2}^{h/2} \frac{zE(z)}{1-\nu^2}\,\mathrm{d}z$$

$$\beta_3 = \int_{-h/2}^{h/2} T(z)E(z)\alpha(z)\,\mathrm{d}z, \qquad \beta_4 = \int_{-h/2}^{h/2} G(z)\,\mathrm{d}z$$

$$\beta_5 = \int_{-h/2}^{h/2} zG(z)\,\mathrm{d}z, \qquad \beta_6 = \int_{-h/2}^{h/2} \frac{z^2E(z)}{1-\nu^2}\,\mathrm{d}z$$

$$\beta_7 = \int_{-h/2}^{h/2} T(z)E(z)\alpha(z)z\,\mathrm{d}z, \qquad \beta_8 = \int_{-h/2}^{h/2} z^2G(z)\,\mathrm{d}z \tag{10}$$

The functional of total potential energy, including the membrane, bending, and thermal strain energies, is written, and the Euler equation is applied to the functional of energy to obtain its stationary value. The result corresponds to the improved nonlinear Donnell equilibrium equations as

$$
\begin{aligned}
&RN_{x,x} + N_{x\theta,\theta} = 0\\
&RN_{x\theta,x} + N_{\theta,\theta} + (1/R)M_{\theta,\theta} + M_{x\theta,x} - (N_\theta\beta_\theta + N_{x\theta}\beta_x) = 0\\
&RM_{x,xx} + 2M_{x\theta,x\theta} + M_{\theta,\theta\theta/R} - N_\theta\\
&\qquad - [RN_x\beta_{x,x} + (\beta_{x,\theta} + R\beta_{\theta,x})N_{x\theta} + N_\theta\beta_{\theta,\theta}] = 0
\end{aligned}
\tag{11}
$$

Here, $\beta_x = -w_{,x}$ and $\beta_\theta = (v - w_{,\theta})/R$ are the rotations in the x and θ directions, respectively. The first and third equations of the system of Eqs. (11) are identical with the Donnell equilibrium equations in terms of force and moment resultants, whereas the second equation is different. Terms $1/RM_{\theta\theta}$, $M_{x\theta,x}$, and $(N_\theta\beta_\theta + N_{x\theta}\beta_x)$ are added to the conventional Donnell equations. The reason is that v is ignored in the expression for β_θ in the Donnell equations. The presence of these terms indicates that the rotations along the x and θ directions and the transverse shear force in θ directions are included in the equilibrium equations. It is customary to neglect these terms in the equilibrium equations. The result of ignoring the terms provides justified buckling loads for the short cylindrical shells. For the long cylindrical shells, the elimination of these terms results in significant error in the buckling load estimation.

The stability equations are obtained by consideration of the second variation of the total functional of strain energy or the force summation method. The displacement components are related to the terms representing the stable equilibrium and the terms of neighboring state. Accordingly, the forces N_{ij} and the moments M_{ij} are divided in two terms representing the stable equilibrium and the neighboring state. Through the linear strain–displacement relations, the expression for total potential function is obtained. This expression, via the Taylor expansion, results in the sum of first and second variations of the total potential energy. Applying the Euler equation to the second variation of the total potential energy function results in the stability equations

$$
\begin{aligned}
&RN_{x_1,x} + N_{x\theta_1,\theta} = 0\\
&RN_{x\theta_1,x} + N_{\theta_1,\theta} + (1/R)M_{\theta_1,\theta} + M_{x\theta_1,x} - \left(N_{\theta_0}\beta_{\theta_1} + N_{x\theta_0}\beta_{x_1}\right) = 0\\
&RM_{x_1,xx} + 2M_{x\theta_1,x\theta} + M_{\theta_1,\theta\theta}\big/R - N_{\theta_1} - RN_{x_0}\beta_{x_1,x}\\
&\qquad - RN_{x\theta_0}\beta_{\theta_1,x} - N_{x\theta_0}\beta_{x_1,\theta} - N_{\theta_0}\beta_{\theta_1,\theta} = 0
\end{aligned}
\tag{12}
$$

In Eqs. (12), terms with the subscript 0 are related to the state of equilibrium and terms with the subscript 1 are those characterizing the state of stability. Note that whereas the equilibrium equations are nonlinear, the stability equations are linear. The terms with the subscript 0 are the solution of the equilibrium equation for the given load. The linearized strains and curvatures in terms of the displacement components are

$$
\begin{aligned}
\epsilon_{xm_1} &= u_{1,x}, \qquad \epsilon_{\theta m_1} = (w_1 + v_{1,\theta})/R\\
\gamma_{x\theta m_1} &= v_{1,x} + (1/R)u_{1,\theta}, \qquad k_{x_1} = -w_{1,xx}\\
k_{\theta_1} &= (v_{1,\theta} - w_{1,\theta\theta})\big/R^2, \qquad k_{x\theta_1} = (v_{1,x} - 2w_{1,x\theta})/2R
\end{aligned}
\tag{13}
$$

The forces and moments associated with the stability state are

$$
\begin{aligned}
N_{x_1} &= \beta_1\epsilon_{xm_1} + \beta_1\nu\epsilon_{\theta m_1} + \beta_2 k_{x_1} + \beta_2\nu k_{\theta_1}\\
N_{\theta_1} &= \beta_1\epsilon_{\theta m_1} + \beta_1\nu\epsilon_{xm_1} + \beta_2 k_{\theta_1} + \beta_2\nu k_{x_1}\\
N_{x\theta_1} &= \beta_4\gamma_{x\theta m_1} + \beta_5 k_{x\theta_1}\\
M_{x_1} &= \beta_2\epsilon_{xm_1} + \beta_2\nu\epsilon_{\theta m_1} + \beta_6 k_{x_1} + \beta_6\nu k_{\theta_1}\\
M_{\theta_1} &= \beta_2\epsilon_{\theta m_1} + \beta_2\nu\epsilon_{xm_1} + \beta_6 k_{\theta_1} + \beta_6\nu k_{x_1}\\
M_{x\theta_1} &= \beta_5\gamma_{x\theta m_1} + \beta_8 k_{x\theta_1}
\end{aligned}
\tag{14}
$$

Substituting Eqs. (13) and (14) in Eqs. (12) gives the stability equations in terms of the displacement components as

$$
\begin{aligned}
&R\beta_1 u_{1,xx} + \frac{\beta_4}{R}u_{1,\theta\theta} + v_{1,\theta x}\left(\beta_1\nu + \underline{\frac{\beta_2\nu}{R}} + \beta_4 + \underline{\frac{\beta_5}{2R}}\right) + \beta_1\nu w_{1,x}\\
&\qquad - R\beta_2 w_{1,xxx} - w_{1,\theta\theta x}\left(\frac{\beta_2\nu}{R} - \frac{\beta_5}{R}\right) = 0
\end{aligned}
\tag{15a}
$$

$$
\begin{aligned}
&\left(\beta_1\nu + \beta_4 + \underline{\frac{\beta_5}{2R} + \frac{\beta_2\nu}{R}}\right)u_{1,\theta x} + \left(\frac{\beta_5}{2} + \underline{\frac{\beta_8}{4R}} + R\beta_4\right)v_{1,xx}\\
&\qquad + \left(\frac{\beta_1}{R} + \frac{\beta_2}{R^2} + \underline{\frac{\beta_6}{R^3}}\right)v_{1,\theta\theta} + w_{1,xx\theta}\left(-\beta_5 - \beta_2\nu - \underline{\frac{\beta_8}{2R}}\right)\\
&\qquad + \left(\frac{\beta_1}{R} + \underline{\frac{\beta_2}{R^2} + \frac{N_{\theta_0}}{R}}\right)w_{1,\theta} - \left(\frac{\beta_2}{R^2} - \underline{\frac{\beta_6}{R^3}}\right)w_{1,\theta\theta\theta} = 0
\end{aligned}
\tag{15b}
$$

$$
\begin{aligned}
&R\beta_2 u_{1,xxx} + \left(\frac{\beta_2\nu}{R} + \frac{\beta_5}{R}\right)u_{1,x\theta\theta} - \beta_1\nu u_{1,x}\\
&\qquad + \left(\beta_2\nu + \underline{\frac{\beta_6\nu}{R}} + \beta_5 + \underline{\frac{\beta_8}{2R}}\right)v_{1,\theta xx} + \left(\frac{\beta_2}{R^2} + \underline{\frac{\beta_6}{R^3}}\right)v_{1,\theta\theta\theta}\\
&\qquad - \left(\frac{\beta_1}{R} + \underline{\frac{\beta_2}{R^2}}\right)v_{1,\theta} + \left(2\beta_2\nu + RN_{x_0}\right)w_{1,xx} + R\beta_6 w_{1,xxxx}\\
&\qquad - \left(\frac{2\beta_6\nu}{R} + \frac{\beta_8}{R}\right)w_{1,\theta\theta xx} + \left(\frac{2\beta_2}{R^2} + \frac{N_{\theta_0}}{R}\right)w_{1,\theta\theta} - \frac{\beta_6}{R^3}w_{1,\theta\theta\theta\theta}\\
&\qquad - \frac{\beta_1}{R}w_1 = 0
\end{aligned}
\tag{15c}
$$

Equations (15) are a coupled set of three partial differential equations for the dependent functions u_1, v_1, and w_1. They are the improved Donnell stability equations in coupled form. The underlined terms in these equations represent the extra terms related to the improved Donnell stability equations. These terms are omitted in the classical Donnell stability equations. Equations (15) are in x, θ, and z directions, respectively.

Consider a cylindrical shell with simply supported edge conditions. The boundary conditions at $x = 0, L$ are[54]

$$
w_1 = w_{1,xx} = v_1 = u_{1,x} = 0 \tag{16}
$$

The one-term approximate solution of the system of Eqs. (15), using the boundary conditions (16), may be assumed as[54]

$$
\begin{aligned}
u_1 &= A_1\cos\lambda x\sin\theta\\
v_1 &= B_1\sin\lambda x\cos n\theta, \qquad 0 \le x \le L\\
w_1 &= C_1\sin\lambda x\sin n\theta, \qquad 0 \le \theta \le 2\pi
\end{aligned}
\tag{17}
$$

where A_1, B_1, and C_1 are constant coefficients and $\lambda = m\pi/L$, where $m = 1, 2, 3, \ldots$, and $n = 1, 2, 3, \ldots$. System (15) is made orthogonal with respect to the approximate solutions (17) according to the Galerkin method. The resulting system of equations with constant coefficients is

$$
\begin{aligned}
&b_{11}A_1 + b_{12}B_1 + b_{13}C_1 = 0, \qquad b_{21}A_1 + b_{22}B_1 + b_{23}C_1 = 0\\
&b_{31}A_1 + b_{32}B_1 + b_{33}C_1 = 0
\end{aligned}
\tag{18}
$$

where

$$b_{11} = \lambda^2 R\beta_1 + \frac{\beta_4 n^2}{R}, \qquad b_{12} = n\lambda\left(\beta_1\nu + \frac{\beta_2\nu}{R} + \beta_4 + \frac{\beta_5}{2R}\right)$$

$$b_{13} = n^2\lambda\frac{\beta_5}{R} - \beta_1\nu\lambda - R\beta_2\lambda^3 - n^2\lambda\frac{\beta_2\nu}{R}$$

$$b_{21} = \lambda n\left(\beta_1\nu + \frac{\beta_5}{2R} + \beta_4 + \frac{\beta_2\nu}{R}\right)$$

$$b_{22} = \lambda^2\beta_5 + \frac{\lambda^2\beta_8}{4R} + \lambda^2 R\beta_4 + \frac{2n^2\beta_2}{R^2} + \frac{\beta_1 n^2}{R} + \frac{\beta_6 n^2}{R^3}$$

$$b_{23} = -\lambda^2 n\beta_5 - \lambda^2 n\beta_2\nu - \lambda^2 n\frac{\beta_8}{2R} - n\frac{\beta_1}{R} - \frac{n\beta_2}{R^2} - \frac{n^3\beta_2}{R^2} - \frac{n^3\beta_6}{R^3}$$

$$b_{31} = R\beta_2\lambda^3 + \frac{\lambda n^2\beta_2\nu}{R} + \frac{\lambda n^2\beta_5}{R} + \beta_1\nu\lambda$$

$$b_{32} = \lambda^2 n\beta_2\nu + \frac{\lambda^2 n\beta_6\nu}{R} + \lambda^2 n\beta_5 + \frac{\lambda^2 n\beta_8}{2R} + \frac{n^3\beta_2}{R^2} + \frac{n^3\beta_6}{R^3} + \frac{n\beta_1}{R} + \frac{n\beta_2}{R^2}$$

$$b_{33} = R\beta_6\lambda^4 - 2\lambda^2\beta_2\nu - \lambda^2 R N_{x_0} - \frac{2\lambda^2 n^2\beta_6\nu}{R} - \frac{\lambda^2 n^2\beta_8}{R} - \frac{2n^2\beta_2}{R^2} - \frac{n^4\beta_6}{R^3} - \frac{\beta_1}{R} \tag{19}$$

Substituting Eqs. (3) in Eqs. (10) gives

$$\beta_1 = \frac{h}{2(1-\nu^2)}(E_c + E_m), \qquad \beta_2 = \frac{h^2}{12(1-\nu^2)}(E_c - E_m)$$

$$\beta_3 = T\left[E_m\alpha_m h + \tfrac{1}{2}E_m\alpha_{cm}h + \tfrac{1}{2}E_{cm}\alpha_m h + \tfrac{1}{3}E_{cm}\alpha_{cm}h\right]$$

$$\beta_4 = \frac{h}{24(1+\nu)}(E_c + E_m), \qquad \beta_5 = \frac{h^2}{24(1+\nu)}(E_c - E_m)$$

$$\beta_6 = \frac{h^3}{24(1-\nu^2)}(E_c - E_m)$$

$$\beta_7 = T\left[\frac{E_m\alpha_{cm}h^2}{12} + \frac{E_{cm}\alpha_m h^2}{12} + \frac{E_{cm}\alpha_{cm}h^2}{12}\right]$$

$$\beta_8 = \frac{h^3}{48(1+\nu)}(E_c + E_m) \tag{20}$$

In Eqs. (20), the coefficients β_3 and β_7 are obtained for the uniform temperature rise, where T is a constant value through the thickness. When T is a function of the radius coordinate, β_3 and β_7 are obtained from Eqs. (10). Equations (20) are substituted in Eqs. (19), and the members of determinant of the matrix of coefficients become

$$b_{11} = \left(\frac{E_c + E_m}{2}\right)\left(\frac{\lambda^2 Rh}{1-\nu^2} + \frac{hn^2}{12R(1+\nu)}\right)$$

$$b_{12} = \left(\frac{E_c + E_m}{2}\right)\left(\frac{h\nu n\lambda}{1-\nu^2} + \frac{hn\lambda}{12(1+\nu)}\right) + \left(\frac{E_c - E_m}{12}\right)\left(\frac{h^2\nu n\lambda}{R(1-\nu^2)} + \frac{h^2 n\lambda}{4R(1+\nu)}\right)$$

$$b_{13} = -\left(\frac{E_c + E_m}{2}\right)\frac{h\nu\lambda}{1-\nu^2} + \left(\frac{E_c - E_m}{12}\right)\left(\frac{h^2 n^2\lambda}{2R(1+\nu)} - \frac{h^2\lambda^3 R}{1-\nu^2} - \frac{h^2 n^2\lambda\nu}{R(1-\nu^2)}\right)$$

$$b_{21} = \left(\frac{E_c + E_m}{2}\right)\left(\frac{h\nu n\lambda}{1-\nu^2} + \frac{h\lambda n}{12(1+\nu)}\right) + \left(\frac{E_c - E_m}{12}\right)\left(\frac{h^2\lambda n}{4R(1+\nu)} + \frac{h^2\nu\lambda n}{R(1-\nu^2)}\right)$$

$$b_{22} = \left(\frac{E_c + E_m}{2}\right)\left(\frac{hR\lambda^2}{12(1+\nu)} + \frac{hn^2}{R(1-\nu^2)} + \frac{h^3\lambda^2}{96R(1+\nu)}\right) + \left(\frac{E_c - E_m}{6}\right)\left(\frac{n^2h^2}{R^2(1-\nu^2)} + \frac{h^2\lambda^2}{4(1+\nu)} + \frac{h^3 n^2}{4R^3(1-\nu^2)}\right)$$

$$b_{23} = -\left(\frac{E_c + E_m}{2}\right)\left(\frac{h^3\lambda^2 n}{48R(1+\nu)} + \frac{hn}{R(1-\nu^2)}\right) - \left(\frac{E_c - E_m}{12}\right)\left(\frac{h^2n^3}{R^2(1-\nu^2)} + \frac{h^2\lambda^2 n\nu}{1-\nu^2} + \frac{h^2\lambda^2 n}{2(1+\nu)} + \frac{h^2 n}{R^2(1-\nu^2)} + \frac{h^3 n^3}{2R^3(1-\nu^2)}\right)$$

$$b_{31} = \left(\frac{E_c + E_m}{2}\right)\frac{h\nu\lambda}{1-\nu^2} + \left(\frac{E_c - E_m}{12}\right)\left(\frac{h^2\lambda^3 R}{(1-\nu^2)} + \frac{h^2 n^2\lambda\nu}{R(1-\nu^2)} + \frac{h^2\lambda n^2}{2R(1+\nu)}\right)$$

$$b_{32} = \frac{E_c + E_m}{2}\left[\frac{h^3\lambda^2 n}{48R(1+\nu)} + \frac{hn}{R(1-\nu^2)}\right] + \left(\frac{E_c - E_m}{12}\right)\left[\frac{h^2\lambda^2 n\nu}{1-\nu^2} + \frac{h^3\lambda^2 n\nu}{2R(1-\nu^2)} + \frac{h^2\lambda^2 n}{2(1+\nu)} + \frac{h^2 n^3}{R^2(1-\nu^2)} + \frac{h^3 n^3}{2R^3(1-\nu^2)} + \frac{h^2 n}{R^2(1-\nu^2)}\right]$$

$$b_{33} = -\left(\frac{E_c + E_m}{2}\right)\left[\frac{h}{R(1-\nu^2)} + \frac{h^3\lambda^2 n^2}{24R(1+\nu)}\right] + \left(\frac{E_c - E_m}{6}\right)\left[\frac{h^3\lambda^4 R}{4(1-\nu^2)} - \frac{h^2\lambda^2\nu}{1-\nu^2} - \frac{h^3\,\lambda^2 n^2\nu}{2R(1-\nu^2)} - \frac{h^2 n^2}{R^2(1-\nu^2)} - \frac{h^3 n^4}{4R^3(1-\nu^2)}\right] - \lambda^2 R N_{x_0} \tag{21}$$

Equations (18) are a system of homogeneous equations that have a nontrivial solution only for discrete values of the thermal load ΔT. The determinant of the coefficients A_1, B_1, and C_1 are set equal to zero, as

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 0 \tag{22}$$

The resulting equation may be solved for the buckling load. The critical buckling load is the minimum value of the determinant for values of m and n, the longitudinal and circumferential buckling waves.

III. Uniform Temperature Rise

The initial uniform temperature of the shell is assumed to be T_i. Under simply supported boundary conditions, temperature is uniformly raised to a final value T_f such that the shell buckles.

To find the critical $\Delta T = T_f - T_i$, the prebuckling forces should be found. Solving the membrane form of the equilibrium equations, using the method developed by Meyers and Hyer,[55] gives the prebuckling force resultants as

$$N_{x_0} = -[\Delta T/(1-\nu)]\big(E_m\alpha_m h + \tfrac{1}{2}E_m\alpha_{cm}h + \tfrac{1}{2}E_{cm}\alpha_m h + \tfrac{1}{3}E_{cm}\alpha_{cm}h\big)$$
$$N_{\theta_0} = N_{x\theta_0} = 0 \tag{23}$$

When coefficients from Eqs. (21) are substituted into Eq. (22) and the result is simplified, the final expression for ΔT becomes

$$\Delta T = \frac{(1-\nu)(b_{11}b_{23}b_{32} + b_{13}b_{22}b_{31} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32})}{\lambda^2 R\big(E_m\alpha_m h + \frac{1}{2}E_m\alpha_{cm}h + \frac{1}{2}E_{cm}\alpha_m h + \frac{1}{3}E_{cm}\alpha_{cm}h\big)(b_{11}b_{22} - b_{12}b_{21})} - \frac{(1-\nu)}{\lambda^2 R\big(E_m\alpha_m h + \frac{1}{2}E_m\alpha_{cm}h + \frac{1}{2}E_{cm}\alpha_m h + \frac{1}{3}E_{cm}\alpha_{cm}h\big)}$$
$$\times\left\{-\left(\frac{E_c+E_m}{2}\right)\left[\frac{h}{R(1-\nu^2)} + \frac{h^3\lambda^2 n^2}{24R(1+\nu)}\right] + \left(\frac{E_c-E_m}{6}\right)\left[\frac{h^3\lambda^4 R}{4(1-\nu^2)} - \frac{h^2\lambda^2\nu}{1-\nu^2} - \frac{h^3\lambda^2 n^2\nu}{2R(1-\nu^2)} - \frac{h^2n^2}{R^2(1-\nu^2)} - \frac{h^3n^4}{4R^3(1-\nu^2)}\right]\right\} \tag{24}$$

The uniform buckling temperature difference is obtained by minimizing Eq. (24) with respect to m and n, the number of longitudinal and circumferential buckling waves. For pure isotropic metal or pure isotropic ceramic in relations (10), $\beta_2 = 0$, $\beta_5 = 0$, and $\beta_7 = 0$ and Eq. (24) reduces to $\Delta T_{\text{cr}} = 0.316h/R\alpha$. This result is given by Eslami et al.[15]

IV. Linear Temperature Change Through the Thickness

Because the shell thickness is thin, the temperature variation along the radial direction through the thickness may be approximated by a linear function as

$$T(z) = (\Delta T/h)(z + h/2) + T_a \tag{25}$$

where $\Delta T = T_b - T_a$, where T_a is temperature of inner surface and T_b is temperature of the outer surface of functionally graded shell. The membrane solution of the equilibrium equations, using the method developed by Meyers and Hyer,[55] results in the prebuckling force resultants

$$N_{x_0} = [-\Delta T/2(1-\nu)]\big(E_m\alpha_m h + \tfrac{1}{3}E_m\alpha_{cm}h + \tfrac{1}{3}E_{cm}\alpha_m h + \tfrac{1}{6}E_{cm}\alpha_{cm}h\big)$$
$$N_{\theta_0} = N_{x\theta_0} = 0 \tag{26}$$

When coefficients from Eqs. (21) are substituted in Eq. (22) and the result is simplified, the final expression for ΔT becomes

$$\Delta T = \frac{2(1-\nu)(b_{11}b_{23}b_{32} + b_{13}b_{22}b_{31} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32})}{\lambda^2 R\big(E_m\alpha_m h + \frac{1}{3}E_m\alpha_{cm}h + \frac{1}{3}E_{cm}\alpha_m h + \frac{1}{6}E_{cm}\alpha_{cm}h\big)(b_{11}b_{22} - b_{12}b_{21})} - \frac{2(1-\nu)}{\lambda^2 R\big(E_m\alpha_m h + \frac{1}{3}E_m\alpha_{cm}h + \frac{1}{3}E_{cm}\alpha_m h + \frac{1}{6}E_{cm}\alpha_{cm}h\big)}$$
$$\times\left\{-\left(\frac{E_c+E_m}{2}\right)\left[\frac{h}{R(1-\nu^2)} + \frac{h^3\lambda^2 n^2}{24R(1+\nu)}\right] + \left(\frac{E_c-E_m}{6}\right)\left[\frac{h^3\lambda^4 R}{4(1-\nu^2)} - \frac{h^2\lambda^2\nu}{1-\nu^2} - \frac{h^3\lambda^2 n^2\nu}{2R(1-\nu^2)} - \frac{h^2n^2}{R^2(1-\nu^2)} - \frac{h^3n^4}{4R^3(1-\nu^2)}\right]\right\} \tag{27}$$

The linear buckling temperature difference is obtained by minimizing Eq. (27) with respect to m and n, the number of longitudinal and circumferential buckling waves. For pure isotropic metal or pure isotropic ceramic in relations (10), $\beta_2 = 0$, and $\beta_5 = 0$, and Eq. (27) reduces to $\Delta T_{\text{cr}} = 0.632h/R\alpha$. This result is obtained and given by Eslami et al.[15]

V. Nonlinear Temperature Distribution Through the Thickness

A cylindrical shell with inner surface temperature T_a and outer surface temperature T_b is considered. Temperature distribution across the thickness is obtained by solving the differential equation of thermal conduction as

$$\frac{\partial^2 T(r)}{\partial r^2} + \left[\frac{1}{K(r)}\frac{\partial K(r)}{\partial r} + \frac{1}{r}\right]\frac{\partial T(r)}{\partial r} = 0 \tag{28}$$

where $K(r)$ is the coefficient of thermal conduction of the functionally graded cylinder. We may assumed a linear distribution for $K(r)$ in the radial direction, as discussed in the Appendices, by

$$K(r) = K_0 r \tag{29}$$

where K_0 is a constant value. The solution of Eq. (28) is obtained by assuming a distribution for temperature of the form

$$T(r) = b_1 r^{t_1} + b_2 r^{t_2} + b_3 \tag{30}$$

where b_1, b_2, b_3, t_1, and t_2 are constant coefficients. Substituting the solution given by Eq. (30) in Eq. (28) and solving for $T(r)$ gives

$$T(r) = b_4/r + b_3 \tag{31}$$

To obtain the constant coefficients b_3 and b_4, the boundary conditions are assumed as

$$T = T_a \quad \text{at} \quad r = r_a$$
$$T = T_b \quad \text{at} \quad r = r_b \tag{32}$$

where r_a and r_b are the inner and outer radii of the cylindrical shell, respectively. When the boundary conditions are used, the temperature distribution across the thickness of the cylindrical shell becomes

$$T(r) = \frac{(T_a - T_b)r_a r_b}{hr} - (T_a - T_b)\frac{r_b}{h} + T_a \tag{33}$$

The prebuckling forces are obtained by solving the membrane form of the equilibrium equations, using the method developed by Meyers and Hyer.[55] The prebuckling force resultants are then

$$
\begin{aligned}
N_{x0} = & \frac{-2A'E_m\alpha_m}{1-\nu}\ln\frac{r_{av}+h/2}{r_{av}} - \frac{A'E_m\alpha_{cm}}{1-\nu}\ln\frac{r_{av}+h/2}{r_{av}} \\
& - \frac{A'E_{cm}\alpha_m}{1-\nu}\ln\frac{r_{av}+h/2}{r_{av}} - \frac{A'E_{cm}\alpha_{cm}}{h^2(1-\nu)}\left[\frac{h^2}{4} - hr_{av}\right. \\
& \left. + 2r_{av}^2\ln\frac{r_{av}+h/2}{r_{av}}\right] - \frac{A'E_{cm}\alpha_{cm}}{2(1-\nu)}\ln\frac{r_{av}+h/2}{r_{av}} - \frac{B'E_m\alpha_m h}{1-\nu} \\
& - \frac{B'E_m\alpha_{cm}h}{2(1-\nu)} - \frac{B'E_{cm}\alpha_{cm}h}{12(1-\nu)} - \frac{B'E_{cm}\alpha_m h}{2(1-\nu)} - \frac{B'E_m\alpha_{cm}h}{4(1-\nu)}
\end{aligned} \tag{34}
$$

where

$$
A' = (T_a - T_b)(r_b r_a/h), \qquad B' = T_a - (T_a - T_b)(r_b/h) \tag{35}
$$

The coefficients from Eqs. (21) are substituted in Eq. (22), and upon simplification, the final expressions for N_{x0} becomes

$$
\begin{aligned}
N_{x0} = & \frac{b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31} - b_{11}b_{23}b_{32}}{\lambda^2 R(b_{12}b_{21} - b_{11}b_{22})} \\
& + \frac{1}{\lambda^2 R}\left\{-\left(\frac{E_c+E_m}{2}\right)\left[\frac{h}{R(1-\nu^2)} + \frac{h^3\lambda^2 n^2}{24R(1+\nu)}\right]\right. \\
& + \left(\frac{E_c-E_m}{6}\right)\left[\frac{h^3\lambda^4 R}{4(1-\nu^2)} - \frac{h^2\lambda^2\nu}{1-\nu^2} - \frac{h^3\lambda^2 n^2\nu}{2R(1-\nu^2)}\right. \\
& \left.\left. - \frac{h^2 n^2}{R^2(1-\nu^2)} - \frac{h^3 n^4}{4R^3(1-\nu^2)}\right]\right\}
\end{aligned} \tag{36}
$$

The buckling force is obtained by minimizing Eq. (36) with respect to m and n, the number of longitudinal and circumferential buckling waves. Then, this expression for N_{x0} is substituted in Eq. (34) to provide the buckling temperature difference $\Delta T_{cr} = T_b - T_a$.

VI. Results and Discussion

Consider a ceramic–metal functionally graded cylindrical shell (Fig. 1) The combination of materials consist of steel and alumina. Young's modulus and the coefficient of thermal expansion are for steel, $E_m = 200$ GPa and $\alpha_m = 11.7 \times 10^{-6}$ 1/°C, and for alumina, $E_c = 380$ GPa and $\alpha_c = 7.4 \times 10^{-6}$ 1/°C, respectively. Poisson's ratio is assumed to be 0.3 for steel and alumina. Simply supported boundary conditions are assumed. The variation of the buckling temperature difference ΔT_{cr} vs the variation of the dimensionless geometrical parameter h/R is plotted for three loading cases in Figs. 2–4. In Figs. 2–4, three arbitrary values of L/R are considered, and the mean radius is assumed $R = 0.5$ m. As can be seen, ΔT_{cr} is increased as L/R and h/R are increased. The buckling temperatures related to the uniform temperature rise and linear temperature through the thickness for cylindrical shells made of isotropic material are higher than the FGM cylindrical shell (Figs. 2 and 3). Comparison of Figs. 3 and 4 reveals that the buckling temperature associated with the nonlinear temperature distribution across the thickness is greater than the linear temperature distribution.

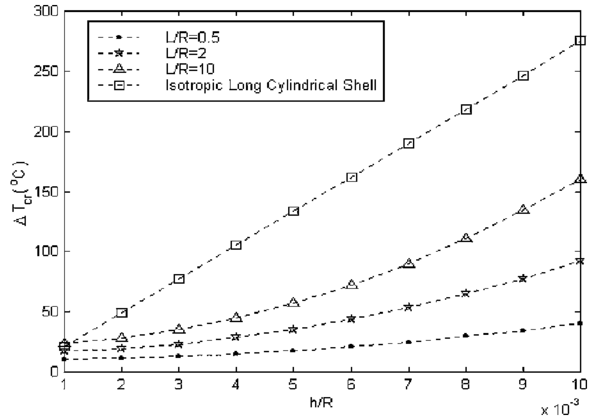


Fig. 2 Variation of the critical uniform temperature rise vs *h/R*.

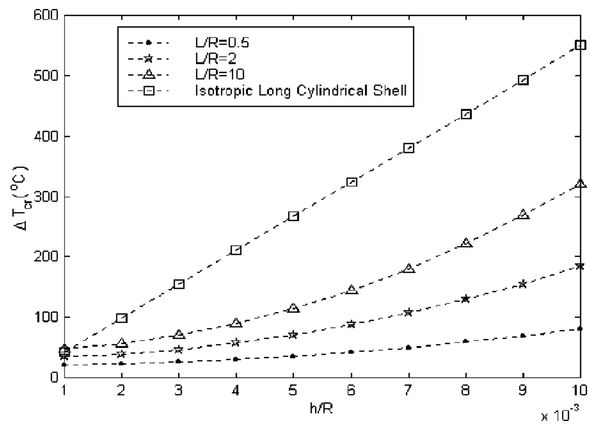


Fig. 3 Variation of the critical linear radial temperature difference vs *h/R*.

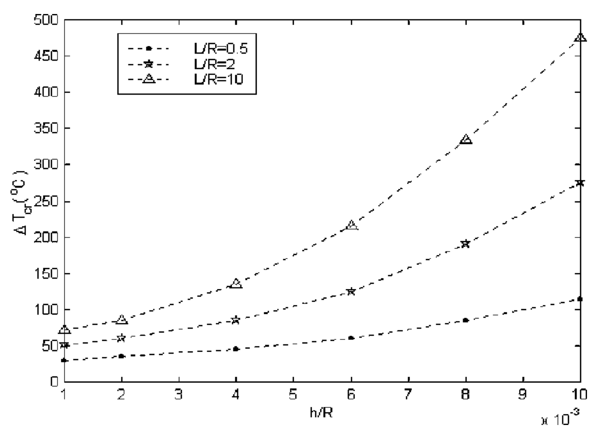
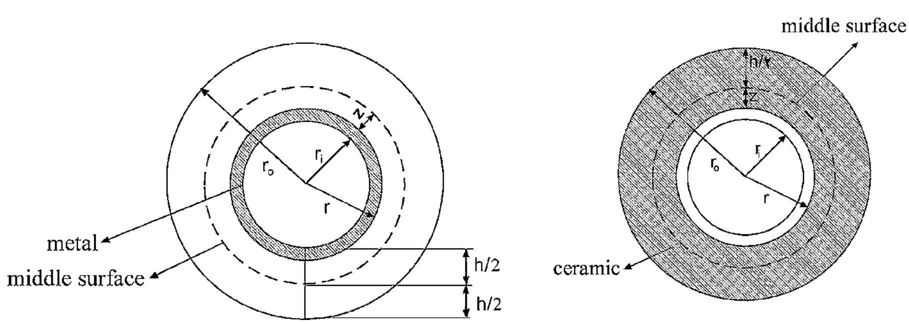


Fig. 4 Variation of the critical nonlinear radial temperature difference vs *h/R*.



a) b)

Fig. 1 Distribution of metal and ceramic in the FGM cylinder.

The buckling modes m and n related to the critical uniform temperature rise are checked. Note that as h/R is increased, assuming constant L/R, the number of buckling waves m and n are decreased. Also, as L/R is increased, with constant h/R, m is increased and n is decreased. The limiting value for m is 1 and for n is 2.

VII. Conclusions

Equilibrium and stability equations for simply supported functionally graded cylindrical shells are obtained. Derivations are based on first-order shell theory the Sanders kinematic relations and the improved Donnell stability equations. The buckling analysis of functionally graded cylindrical shells under three different loading cases are investigated. The following conclusions are reached:

1) The equilibrium and stability equations are identical with the corresponding equations for the pure isotropic cylindrical shell in form of forces and moments per unit length.

2) The critical temperature difference $\Delta T_{\rm cr}$ for the functionally graded cylindrical shell is generally lower than the corresponding value for the pure isotropic cylindrical shell.

3) The critical temperature difference $\Delta T_{\rm cr}$ for the functionally graded cylindrical shell is increase by increasing the dimensionless ratios h/R and L/R.

4) The critical temperature difference $\Delta T_{\rm cr}$ for a FGM cylindrical shell under a linear temperature difference through the thickness is twice of the FGM cylindrical shell under uniform temperature difference.

5) The critical temperature difference $\Delta T_{\rm cr}$ for FGM cylindrical shell under nonlinear temperature difference through the thickness is greater than the FGM cylindrical shell under linear temperature difference through the thickness.

Appendix A: Calculation of Volume Fraction of Metal in Cylinder

Figure 1 shows the distribution of ceramic and metal in the cross section of cylindrical shell. To calculate the volume fraction of metal (Fig. 1a), the volume of metal $\forall_m$ and the total volume $\forall_{m+c}$ must be used as

$$\forall_m = 2\pi r_i (r - r_i) L, \qquad \forall_{m+c} = 2\pi r_i (r_0 - r_i) L \tag{A1}$$

Therefore,

$$f_m = \frac{\forall_m}{\forall_{m+c}} = \frac{2\pi r_i (r - r_i) L}{2\pi r_i (r_0 - r_i) L} = \frac{r - r_i}{r_0 - r_i} = \frac{h/2 - z}{h} = \frac{h - 2z}{2h} \tag{A2}$$

Appendix B: Calculation of Volume Fraction of Ceramic in Cylinder

To calculate the volume fraction of ceramic (Fig. 1b), the volume of ceramic $\forall_c$ and the total volume $\forall_{m+c}$ must be used as

$$\forall_c = 2\pi r_0 (r_0 - r) L, \qquad \forall_{m+c} = 2\pi r_0 (r_0 - r_i) L \tag{B1}$$

Thus,

$$f_c = \frac{\forall_c}{\forall_{m+c}} = \frac{2\pi r_0 (r_0 - r) L}{2\pi r_0 (r_0 - r_i) L} = \frac{r_0 - r}{r_0 - r_i} = \frac{h/2 + z}{h} = \frac{h + 2z}{2h} \tag{B2}$$

With consideration of the Voigt model, the overall mechanical properties are

$$P_t \forall_t = P_m \forall_m + P_c \forall_c \tag{B3}$$

Here, P_t, P_m, and P_c are the mechanical properties of the overall FGM, metal, and ceramic, respectively. The weighted average of the mechanical properties is

$$P_t = P_m f_m + P_c f_c \tag{B4}$$

Substituting relations (A1) and (B2) in relation (B4) gives

$$P_t = P_m[(h - 2z)/2h] + P_c[(h + 2z)/2h] \tag{B5}$$

Therefore,

$$P_t = P_m + P_{cm}[(h + 2z)/2h], \qquad P_{cm} = P_c - P_m \tag{B6}$$

Here, P may be the modulus of elasticity, thermal expansion coefficient, Poisson's ratio, or any other material properties being a function of graded materials.

References

[1] Suresh, S., and Mortensen, A., *Fundamentals of Functionally Graded Materials*, Cambridge Univ. Press, London, 1998.

[2] Fuchiyama, T., Noda, N., Tsuji, T., and Obata, Y., "Analysis of Thermal Stress and Stress Intensity Factor of Functionally Gradient Materials," *Ceramic Transactions, Functionally Gradient Materials*, Vol. 34, 1993, pp. 425–432.

[3] Williamson, R. L., Rabin, B. H., and Drake, J. T., "Finite Element Analysis of Thermal Residual Stresses at Graded Ceramic–Metal Interfaces, Part I. Model Description and Geometric Effects," *Journal of Applied Physics*, Vol. 74, 1993, pp. 1310–1320.

[4] Fukui, Y., Yamanaka, Y., and Wakashima, K., "The Stress and Strains in a Thick-Walled Tube of Functionally Graded Materials Under Uniform Thermal Loading," *International Journal of Japan Society of Mechanical Engineers*, Ser. A, Vol. 36, 1993, pp. 156–162.

[5] Delale, F., and Erdogan, F., "The Crack Problem for a Nonhomogeneous Plane," *Journal of Applied Mechanics*, Vol. 50, 1983, pp. 609–614.

[6] Noda, N., and Jin, Z. H., "Thermal Stress Intensity Factors for a Crack in a Strip of a Functionally Gradient Material," *International Journal of Solids and Structures*, Vol. 30, 1993, pp. 1039–1056.

[7] Araki, N., Makino, A., Ishiguro, T., and Mihara, J., "An Analytical Solution of Temperature Response in Multilayered Material for Transient Method," *International Journal of Thermophysics*, Vol. 13, 1992, pp. 515–538.

[8] Tanigawa, Y., "Theoretical Approach of Optimum Design for a Plate of Functionally Gradient Materials Under Thermal Loading," *Thermal Shock and Thermal Fatigue Behavior of Advanced Ceramics*, NATO ASI Ser. E, Vol. 241, 1992, pp. 171–180.

[9] Zhai, P. C., Jiang, C. R., and Zhang, Q. J., "Application of Three-Phase Micromechanical Theories to Ceramic/Metal Functionally Gradient Materials," *Ceramic Transactions, Functionally Gradient Materials*, Vol. 34, 1993, p. 449.

[10] Nan, C. W., Yuan, R. Z., and Zhang, L. M., "The Physics of Metal/Ceramic Functionally Gradient Materials," *Ceramic Transactions, Functionally Gradient Materials*, Vol. 34, 1993, pp. 75–82.

[11] Pindera, M. J., and Freed, A. D., "The Effect of Matrix Microstructure on the Evolution of Residual Stresses in Titanium–Aluminide Composites," AMD Vol. 40, American Society of Mechanical Engineers, New York, 1992, pp. 37–52.

[12] Pindera, M. J., Salzar, R. S., and Williams, T. O., "An Evaluation of a New Approach for the Thermoplastic Response of Metal–Matrix Composites," *Composites Engineering*, Vol. 3, 1993, pp. 1185–1201.

[13] Aboudi, J., Pindera, M. J., and Arnold, S. M., "Thermoelastic Response of Metal Matrix Composites with Large Diameter Fibers Subjected to Thermal Gradients," NASA TM 106344, April 1993.

[14] Pindera, M. J., Arnold, S. M., Aboudi, J., and Hui, D., "Use of Composites in Functionally Gradient Materials," *Composites Engineering*, Vol. 4, 1994, pp. 154–180.

[15] Eslami, M. R., Ziaii, A. R., and Ghorbanpour, A., "Thermoelastic Buckling of Thin Cylindrical Shells Based on Improved Donnell Equations," *Journal of Thermal Stresses*, Vol. 19, 1996, pp. 299–316.

[16] Eslami, M. R., and Javaheri, R., "Thermal and Mechanical Buckling of Composite Cylindrical Shells," *Journal of Thermal Stresses*, Vol. 22, No. 6, 1999, pp. 527–545.

[17] Eslami, M. R., and Shariyat, M., "A High-Order Theory for Dynamic Buckling and Postbuckling Analysis of Laminated Cylindrical Shell," *Journal of Pressure Vessel Technology*, Vol. 121, Feb. 1999, pp. 94–102.

[18] Eslami, M. R., and Shariyat, M., "Dynamic Buckling and Postbuckling of Imperfect Orthotropic Cylindrical Shells Under Mechanical and Thermal Loads, Based on the Three-Dimensional Theory of Elasticity," *Journal of Applied Mechanics*, Vol. 66, June 1999, pp. 476–484.

[19] Eslami, M. R., and Shariyat, M., "Elastic, Plastic, and Creep Buckling of Imperfect Cylinders Under Mechanical and Thermal Loading," *Journal of Pressure Vessel Technology*, Vol. 119, No. 1, 1997, pp. 27–36.

[20] Eslami, M. R., and Shahsiah, R., "Thermal Buckling of Imperfect Cylindrical Shells," *Journal of Thermal Stresses*, Vol. 24, No. 1, 2000, pp. 71–90.

[21]Birman, V., and Bert, C. W., "Buckling and Post-Buckling of Composite Plates and Shells Subjected to Elevated Temperature," *Journal of Applied Mechanics*, Vol. 60, 1993, pp. 514–519.

[22]Shen, H. S., "Post Buckling Analysis of Imperfect Stiffened Laminated Cylindrical Shells Under Combined External Pressure and Axial Compression," *Computers and Structures*, Vol. 63, 1997, pp. 335–348.

[23]Shen, H. S., "Postbuckling Analysis of Imperfect Stiffened Laminated Cylindrical Shells under Combined External Pressure and Thermal Loading," *International Journal of Mechanical Sciences*, Vol. 40, 1998, pp. 339–355.

[24]Shen, H. S., "Thermomechanical Postbuckling of Composite Laminated Cylindrical Shells with Local Geometric Imperfections," *International Journal of Solids and Structures*, Vol. 36, 1999, pp. 597–617.

[25]Iu, V. P., and Chia, C. Y., "Effect of Transverse Shear on Nonlinear Vibration and Postbuckling of Anti-Symmetric Cross-Ply Imperfect Cylindrical Shells," *International Journal of Mechanical Sciences*, Vol. 30, 1988, pp. 705–718.

[26]Reddy, J. N., and Savoia, M., "Layerwise Shell Theory for Postbuckling of Laminated Circular Cylindrical Shells," *AIAA Journal*, Vol. 30, 1992, pp. 2148–2154.

[27]Whitney, J. M., and Ashton, J. E., "Effect of Environment on the Elastic Response of Layered Composite Plates," *AIAA Journal*, Vol. 9, 1971, pp. 1708–1713.

[28]Snead, J. M., and Palazotto, A. N., "Moisture and Temperature Effects on the Instability of Cylindrical Composite Panels," *Journal of Aircraft*, Vol. 20, 1983, pp. 777–783.

[29]Lee, S. Y., and Yen, W. J., "Hygrothermal Effects on the Stability of a Cylindrical Composite Shell Panel," *Computers and Structures*, Vol. 33, 1989, pp. 551–559.

[30]Ram, K. S. S., and Sinha, P. K., "Hygrothermal Effects on the Buckling of Laminated Composite Plates," *Composite Structures*, Vol. 21, 1992, pp. 233–247.

[31]Chao, L. P., and Shyu, S. L., "Nonlinear Buckling of Fiber Reinforced Composite Plates Under Hygrothermal Effects," *Journal of Chinese Institute of Engineers*, Vol. 19, 1996, pp. 657–667.

[32]Palazotto, A. N., and Tisler, T. W., "Experimental Collapse Determination of Cylindrical Composite Panels with Large Cutouts Under Axial Load," *Composite Structures*, Vol. 12, 1989, pp. 61–78.

[33]Palazotto, A. N., *An Experimental Study of a Curved Composite Panel with a Cutout*, Vol. 972, American Society for Testing and Materials, Philadelphia, 1988, pp. 191–202.

[34]Horban, B. A., and Palazotto, A. N., "Experimental Buckling of Cylindrical Composite Panels with Eccentrically Located Circular Delaminations," *Journal of Spacecraft and Rockets*, Vol. 24, 1987, pp. 349–352.

[35]Siefert, G. R., and Palazotto, A. N., "The Effect of a Centrally Located Midplane Delamination on the Stability of Composite Panels," *Experimental Mechanics*, Vol. 26, 1986, pp. 330–336.

[36]Dennis, S. T., and Palazotto, A. N., "Large Displacement and Rotational Formulation for Laminated Shells Including Parabolic Transverse Shear," *International Journal of Nonlinear Mechanics*, Vol. 25, 1990, pp. 67–85.

[37]Dennis, S. T., and Palazotto, A. N., "Transverse Shear Deformation in Orthotropic Cylindrical Pressure Vessels Using a High-Order Shear Theory," *AIAA Journal*, Vol. 27, 1989, pp. 1441–1447.

[38]Tsai, C. T., Palazotto, A. N., and Dennis, S. T., "Large Rotation Snap Through Buckling in Laminated Cylindrical Panels," *Finite Elements in Analysis and Design*, Vol. 9, 1991, pp. 65–75.

[39]Dennis, S. T., and Palazotto, A. N., "Effect of Nonlinear Curvature Strains on the Buckling of Laminated Plates and Shells," *International Journal for Numerical Methods in Engineering*, Vol. 36, 1993, pp. 595–610.

[40]Schimmels, S. A., and Palazotto, A. N., "Nonlinear Geometric and Material Behavior of Composite Shells with Large Strains," *Journal of Engineering Mechanics*, Vol. 120, 1994, pp. 320–345.

[41]Palazotto, A. N., Chien, L. S., and Taylor, W. W., "Stability Characteristics of Laminated Cylindrical Panels Under Transverse Loading," *AIAA Journal*, Vol. 30, 1992, pp. 1649–1653.

[42]Chien, L. S., and Palazotto, A. N., "Dynamic Buckling of Composite Cylindrical Panels with Higher-Order Transverse Shear Subjected to a Transverse Concentrated Load," *International Journal of Nonlinear Mechanics*, Vol. 27, 1992, pp. 719–734.

[43]Schimmels, S. A., and Palazotto, A. N., "Collapse Characteristics of Cylindrical Panels Under Axial Loads," *AIAA Journal*, Vol. 30, 1992, pp. 1447–1466.

[44]Shen, H. S., "Hygrothermal Effects on the Postbuckling of Composite Laminated Cylindrical Shells," *Composite Science and Technology*, Vol. 60, 2000, pp. 1227–1240.

[45]Birman, V., "Buckling of Functionally Graded Hybrid Composite Plates," *Proceeding of the l0th Conference on Engineering Mechanics*, Vol. 2, Elsevier Science, 1995, pp. 1199–1202.

[46]Ng, T. Y., Lam, Y. K., Liew, K. M., and Reddy, J. N., "Dynamic Stability Analysis of Functionally Graded Cylindrical Shell Under Periodic Axial Loading," *International Journal of Solids and Structures*, Vol. 38, 2001, pp. 1295–1300.

[47]Javaheri, R., and Eslami, M. R., "Thermoelastic Buckling of Rectangular Plates Made of Functionally Graded Materials," *AIAA Journal*, Vol. 40, No. 1, 2002, pp. 162–169.

[48]Javaheri, R., and Eslami, M. R., "Buckling of Functionally Graded Plates under In-plane Compressive Loading," *ZAMM*, Vol. 82, No. 4, 2002, pp. 277–283.

[49]Javaheri, R., and Eslami, M. R., "Buckling of Functionally Graded Plates under In-plane Compressive Loading Based on Various Plate Theories," *Journal of Iranian Society of Mechanical Engineers* (submitted for publication).

[50]Javaheri, R., and Eslami, M. R., "Thermal Buckling of Functionally Graded Plates Based on Higher Order Theory," *Journal of Thermal Stresses*, Vol. 25, No. 7, 2002, pp. 603–625.

[51]Najafizadeh, M. M., and Eslami, M. R., "First-Order-Theory-Based Thermoelastic Stability of Functionally Graded Material Circular Plates," *AIAA Journal*, Vol. 40, No. 7, 2002, pp. 444–450.

[52]Praveen, G. N., and Reddy, J. N., "Nonlinear Transient Thermoelastic Analysis of Functionally Graded Ceramic Metal Plates," *International Journal of Solids and Structures*, Vol. 35, No. 33, 1998, pp. 4457–4476.

[53]Sugano, Y., "An Analytical Solution for a Plane Thermal Stress Problem in Nonhomogeneous Multiply Connected Regions," *Japan Society of Mechanical Engineers International Journal*, Ser. A, Vol. 33, 1990, pp. 136–144.

[54]Brush, D. O., and Almroth, B. O., *Buckling of Bars, Plates and Shells*, McGraw–Hill, New York, 1975.

[55]Meyers, C. A., and Hyer, M. W., "Thermal Buckling and Postbuckling of Symmetrically Laminated Composite Plates," *Journal of Thermal Stresses*, Vol. 14, 1991, pp. 519–540.

A. N. Palazotto
Associate Editor